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ABSTRACT

This paper considers the relationships between mathematics, creativity, and the socioeconomically disadvantaged teenager. The aim is to ascertain better how to recognize the mathematically gifted student and then to pursue the question of how best to facilitate his or her learning of mathematics. Research and other literature are analyzed, followed by suggestions on approaches to the mathematics curriculum, teaching, and learning. (MK)

MATHEMATICS CREATIVITY AND THE SOCIOECONOMICALLY DISADVANTAGED

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This paper is based on work carried out at Queen's University during the 1978-79 Academic year when the senior author was on leave from the Washington, D.C., Public School system. Comments on and reactions to the paper would be appreciated by either author.

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Once there was a man who set out to find the greatest general who had ever lived. Upon inquiring, he was told that the person he sought had died and gone to heaven. At the Pearly Gates, he informed St. Peter of the purpose of his quest, whereupon St. Peter pointed to a nearby soul. "But," protested the inquirer, "that isn't the greatest of all generals! I knew that person when he lived on earth, and he was only a cobbler." "I know that," replied St. Peter, "but had he been a general, he would have been the greatest of them all."

Mark Twain

Everyone in whom there is a Raphael should have the opportunity to develop himself unimpeded.

Karl Marx

CONTENTS

Note: For convenience, the words he, him, and his have been used throughout this paper to refer to students and teachers despite the writers' conviction that when we write of mathematically gifted teenagers and their teachers at least half of the individuals referred to are women.

To give a fair chance to potential creativity is a matter of life and death for any society...because the outstanding creative ability of a fairly small percentage of the population is mankind's ultimate asset.[1]

Introduction

In this paper, we will look at the relationships between mathematics, creativity, and the socioeconomically disadvantaged teenager in order to see better how to recognize the mathematically gifted student and then to pursue the question of how best to facilitate his learning of mathematics.

Children from low-income ethnic and racial minority groups probably represent America's largest untapped source of talent. Beyond any humanitarian desire to overcome poverty and discrimination, and beyond the moral necessity of providing equality of opportunity, the survival and welfare of mankind depend on our success in nurturing talents of all kinds wherever we may find them.[2]

There are proponents of gifted education who believe that a gifted child with educational inadequacies is a contradiction in terms. We disagree radically with that point of view and insist that there is no basis for not providing the disadvantaged gifted student with special opportunities that are to some extent compensatory in nature.

At the same time, we point out that insofar as the economically disadvantaged child is also culturally different from the mainstream, he has certain advantages over his more affluent peers.

The program we will outline can best be put in practice in separate classrooms; if that, or if spending money on education of gifted students offends someone, let us say that depriving the gifted and talented students of opportunities to develop special gifts and to use them will in no way improve the achievements of their less capable fellow students.

1.1 Mathematics

The general public has very little idea of what mathematics is about or what it is that research mathematicians do. It is commonly believed that mathematics is developed in a cold and logical way by brilliant and unerring researchers.[3] Sadly, the 'general public' here described includes a great many students and teachers of mathematics.

The student of mathematics should be permitted to see not only the beauty and precision of the end product but also the circumstances that led to the investigation, the false starts, the refuted proofs, the less than elegant proofs, the improvements.

Mathematics is the science of structure, the study of patterns and relationships, and it is developed by human creative intelligence willing to take risks. Mathematical 'truth' is arrived at by careful investigation, imaginative leaps, and re-investigation. Imre Lakatos' *Proofs and Refutations* gives a delightful recounting of the historical development of a well-known mathematical problem and helps us to see the method of mathematical discovery and creativity. Lakatos writes, "Mathematics does not grow through a monotonous increase in the number of indubitably established theorems, but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations."[4]

Mathematics is sometimes taught as if it were nothing more than a static collection of facts to be memorized or a set of algorithms to be learned. Mathematics is dynamic and growing. Its structure is beautiful and pleasing to the intellect, its power for problem solving almost certainly unmatched.

More than a set of discrete processes, mathematics is itself a process. The mathematician must be a clear-minded problem recognizer and a creative problem solver as well as being proficient in the separate skills of the art of mathematics. The final work of art, the clear and concise theory that ultimately emerges, should be recognized as a product of the process.

If we see the aesthetic beauty of the structure of mathematics and the marvelous power of the process of mathematics, then we begin to know *what* we want to teach the mathematically talented. And if we understand how mathematics develops and progresses, we begin to see *how* we want to teach the mathematically talented.

1.2 Creativity

Piaget has written, "The principal goal of education is to create men who are capable of doing new things, not simply of repeating what other generations have done--men who are creative, inventive, discoverers. The second goal of education is to form minds which can be critical, can verify, and not accept everything they are offered."[5]

A reasonable judgment based on observation of our schools and not on our stated aims and philosophies might be that the principal goal of education is submissiveness. A creative child must be cajoled or punished into submission before we can teach him what we want him to know. We teach children not to be critical, to accept what their teacher offers without question, and only rarely do we provide opportunity for the child to create, to invent, to discover.[6]

For too long, educators have tried to represent each person's mental abilities by a single index called the IQ (intelligence quotient). This concept has limited thinking concerning some of the most vital problems and has slowed progress toward bringing about a more complete education through which all individuals might have a better opportunity to realize their potential.[7]

It is odd that such a severely restricted concept of intelligence came to be associated with the tests developed by Alfred Binet when he himself adamantly spoke out against methods of education which required the child to use only memory, and never called upon the student to judge anything, reflect upon anything, or to produce anything new. Binet set these goals for developing a child's mind: to produce and test ideas on his own, to act spontaneously, to judge for himself, to explain what he sees, to defend his own ideas, to practice making decisions, to plan his own days, to imagine, to invent, to live on his own account, to feel the excellence and responsibility of free action.[8]

It is easy to understand how, despite Binet's beliefs, his 'intelligence test' became the standard means by which the intellectual abilities of the young were differentiated. It was relatively straightforward to administer, and the results were to be interpreted in an obvious manner by a numerical scale. High numerical results meant high intelligence; low numerical results meant low intelligence. And each answer was either correct or incorrect. There appeared to be a fair degree of validity to the results and they were not too hard to arrive at. When a student got low marks on IQ tests but persisted in doing well in school, he was said to be 'working beyond his ability,' a phrase which does not bear close scrutiny.

But what does an intelligence test actually test? J.P. Guilford, in his seminal paper "Three Faces of Intellect" (*American Psychologist*, 1959, August) has conceptualized the structure of the intellect as a three-dimensional unit. In Guilford's model, there are eighty specific high level talents. Typical intelligence tests cover eight of these: one tenth of those known. Intelligence tests do not test the other nine tenths of the known specific intellectual talents.[9] The most commonly used intelligence tests have been made up primarily of material in the semantic category of the content dimension of Guilford's structure. (Guilford believes that aptitude for mathematics rests heavily on abilities described by the symbolic category.)[10]

Guilford's research on divergent production abilities (creative thinking) has been especially effective in directing educators and psychologists away from their dependence on a single measure of giftedness. Divergent production is the generation of information, from given information, where emphasis is on variety and quantity of output from the same source. We can use the term *creative thinking* to refer to fluency (large number of ideas), flexibility (variety of different approaches), originality, elaboration (well-developed and detailed ideas), sensitivity to defects and problems, redefinition (perceiving in a way different from the usual and established way).

In 1962, Getzels and Jackson published *Creativity and Intelligence, Explorations with Gifted Students*, in which they showed that highly creative adolescents or adolescents with outstanding divergent thinking ability, achieved as well as their highly intelligent (by IQ measure) peers, in spite of the fact that their average IQ was twenty-three points lower. Paul Torrance and his associates have replicated the Getzels/Jackson study with exactly the same findings in two thirds of the schools studied. In the other one third, they found, however, that the high IQ group scored higher on tests of achievement than did the high creative group. Torrance asserts that in these schools the highly creative individuals were given very little opportunity to use their creative thinking abilities in acquiring educational skills; in these schools children were taught in a highly authoritative manner.[11]

There are at least two lessons for us in these facts. First, we must recognize that if we are looking for gifted learners, learners who have the ability to excel scholastically and to make outstanding contributions to society, then we should look not only at those individuals who seem to be highly intelligent but also at those who, despite closer to average "intelligence," seem to be highly creative. (The quotation marks indicate an objection to the definition of high intelligence as the intelligence of one who scores high on an IQ test. It has been generally agreed that the standard IQ tests discriminate against culturally different individuals; it is likely that they also discriminate against individuals with a highly developed divergent thinking capacity.)

The second lesson we should learn from the Getzels/Jackson study and the Torrance follow-up studies is that giftedness must be nurtured if it is to result in high achievement. If a person's outstanding abilities are in producing original ideas, drawing conclusions from a great assortment of ideas, defending his own ideas, inventing, and independence, but the great preponderance of his education ignores those talents and concentrates instead on memorization, calculation by given algorithms, and reproduction of certain set problems, then should we be surprised if after ten years or so of such education this person, despite his outstanding potential, achieves only as the average learner?

IQ measures do select many students who are likely to achieve better than average, but they also 'select out' some students who are likely to achieve better than average. Research has shown a positive relationship between tests of creativity and tests of scholastic achievement.[12] Thus we must certainly include measures of creativity in our methods of search for gifted students.

The cognitive functioning of the creative individual is characterized by a cognitive preference for complexity (the rich, dynamic, and asymmetrical), cognitive flexibility, and perceptual openness (a greater awareness of and receptiveness to both the outer world and the inner self). Some studies suggest that included in the greater cognitive flexibility of the highly creative person is a great willingness to integrate non-conscious material with conscious material. The imaginative production of the adolescents in the Getzels/Jackson study as shown by their stories and drawings suggested that they used pre-conscious material in conscious expression and that the preconscious was under ego control as evidenced by the uniqueness of their responses and their adaptiveness to reality.[13]

The creative personality is marked by impulsivity, independence, introversion, intuitiveness, self-acceptance, unconventionality, and openness to feelings.[14] From Carl Rogers' point of view, the creative individual's creativity is an attempt to realize and complete himself, 'to become his potentialities.'[15]

1.3 Socioeconomically Disadvantaged Youth

Not everyone who is economically disadvantaged is socially disadvantaged (and not everyone who is socially disadvantaged is economically disadvantaged). However, economic need is likely to create a social disadvantage.

An immediate effect of socioeconomic disadvantage is poor nutrition and limited stimulation in infancy and early childhood. Parents' time with children is likely to be minimal; hence the child's experiences and environment are likely to be limited. Thus, the urban poor, though so close to the intellectual stimulation of the city, may grow up without making use of the public libraries, or the museums, without even seeing the beautiful architecture outside their own neighborhoods, without attending the plays and concerts presented, often free, for children.

The rural poor, on the other hand, are likely to have much more of the time and care of parents, but resources are so much less readily available that only an unusual parent will provide very much of the intellectual stimulation his child would profit from.

A second source of trouble for urban youth in economically deprived circumstances is the tremendous amount of stress that so frequently besets their lives and inadequate opportunity for its relief.

School counselors usually neglect individuals exposed to chronically stressful conditions at home, in the neighborhood, and at school, yet these conditions, much more than brief traumatic experiences, are the ones that lead to personality breakdown.[16]

If a young person suffering from severe stress is unable to channel his reactions into creative channels, he may suffer serious personal damage and may respond negatively with violence of some kind.

But if the opportunity for creativity exists, the personal resources of the individual may allow him to deal creatively with the stress and by overcompensation permit a time of great stress to lead to a time of unusual productivity and creativity. Research into the lives of adults under stress has shown case after case of this phenomenon (for example, following the breakup of a marriage).[17] But for this to occur, the individual must be in an environment that permits him to create. He might want to throw himself into the discovery of mathematics or into some major mathematical project; we must be sure that the way is open.

The urban youth is often at a disadvantage in language skills. Much has been written about the non-standard English used in American subcultures, e.g., "The Logic of Non-Standard English" by W. Labor in *Language, Society, and Education: A Profile of Black English,*[18] and we are beginning to accept and respect these languages. However, children whose languages have not been accepted in school may find it unusually difficult to learn to function adequately with the standard language; similarly, children who simply have not been taught to use and understand standard English are at an obvious disadvantage in advanced learning in any area.

Perhaps the greatest disadvantage to be overcome is the state of the schools attended by the economically deprived child. Our educational practices and our socializing practices reinforce the inequalities of class by limiting access to knowledge by the poor while facilitating access by those who are better off. "Our practice of education, both in and out of school, assures uneven distribution not only of knowledge but also of competence to benefit from knowledge...by limiting and starving the capabilities of the children of the poor".[19]

In our inner city schools, average drops in measured intelligence of black children of twenty points has been recorded as they progress through elementary schools.[20] Even while we recognize the likelihood of a culture bias in the IQ tests, we find it notable that the longer a child spends in the supposedly acculturating institution, the school, the worse he fares on measures of 'intelligence' standardized in the mainstream culture.

As to numbers of gifted children in disadvantaged situations, we certainly have no statistics at this time; but Lewis Terman's famous study of gifted children showed that in actual numbers the non-professional segment of the general population contains more than twice as many gifted children as the professional segment.[21]

For the creative person, it is not enough that problems be solved; there is a further demand that the solutions be elegant. He seeks both truth and beauty.[22]

2.1 Mathematics and Creativity

Is creativity a desirable trait in a mathematician? Obviously yes! A mathematician who simply learns what others have done and reproduces it does indeed have a skill to offer society: he solves problems (which have essentially already been solved) in situations where the solutions have applications for people who do not already know the solution. He may also help to disseminate the accumulated knowledge of mathematics. But a creative mathematician, one who can see deeply into as yet unsolved problems, or one who invents or recognizes new problems has a much rarer talent; without such creativity, we make no progress.

We need divergent thinking to make progress. Sometimes the solution to a problem comes when we connect several pieces of information that had simply never been considered together before; it takes creativity to draw them together. Sometimes intriguing and highly useful theories arise out of investigations that seem altogether frivolous to the conservative mathematicians of the day (witness Georg Cantor and the theory of sets); it takes creativity to develop a new set of principles or a new environment for old principles.

What are some of the many things we might mean by a mathematically gifted child?

1. A child who masters the idea of number and facility with basic operations on numbers at a very early age with minimal assistance.
2. A child who, by means often unclear to an observer, solves problems quickly and easily.
3. A child who solves problems creatively and demonstrates an interest in the process as well as the solution.
4. A child who enjoys mathematics and looks for math-related books to read and problems to solve.

5. A child who does not appear to enjoy mathematics or to seek involvement with it, but who scores high on the mathematics scales in achievement tests and aptitude tests.

6. A child who always earns very high grades in school mathematics courses.

7. A child who invents problems and puzzles.

We could continue the list, but let us pause to look at some of the attributes listed. Items 2 and 3 describe a creative child. The child in item 2 is a divergent thinker who may choose non-standard problem solving methods. Such a child might not respond well to demands to do the problem the 'right' way, and his previous experience with adults, or with mathematics teachers in particular, may prohibit his explaining that his way *is* right. Such a child should be encouraged to defend his thinking; he should be praised for coherent clarification of his methods if that is forthcoming, and, if not, his inability to give a clear explanation should be met with patience (and assistance in expressing his thinking).

The child in item 3 is easier to recognize as gifted, and we are in less danger of putting out the fire. But again, he must have our respect. If his solution to a problem is not precisely what we expected, we must endeavor to see the value in his approach.

The child in item 5 clearly has talent but has perhaps never been presented with a challenging or interesting mathematics problem, has perhaps never been in the company of someone who appreciated the beauty of mathematics, has perhaps never seen a really beautiful mathematics book. If his environment does not offer him something to excite his interest in mathematics, he will have no interest in developing his talent.

Item 7 describes another creative child, someone who does not simply answer the questions set before him but who sees what questions can be asked about a set of data. If he asks questions he cannot answer, he may be led either to see why they are unanswerable or to learn techniques or methods of investigation that would help him to answer them.

Creativity is an essential ingredient in the best mathematical minds. Mathematics is more than a cut-and-dried set of material to be learned by the industrious student. It is a creative act, and like a painting or a sculpture, each such creative act will be judged by its beauty, its mathematical elegance, perhaps to be improved upon or radically changed.

V.A. Krutetskii lists the "component mathematical abilities that arise from the basic characteristics of mathematical thought":[23]

1. An ability to formalize mathematical materials, to isolate form from content, to operate with formal structures of relationships and connections.

2. An ability to generalize mathematical material, to detect what is most important, ignoring the irrelevant, to see what is common and what is different.

3. An ability to operate with numerals and other symbols.

4. An ability for sequential, properly segmented logical reasoning which is related to the need for proof, substantiation, and deductions.

5. An ability to shorten the reasoning process, to think in curtailed structures.

6. An ability to reverse a mental process.

7. Flexibility of thought; an ability to switch from one mental operation to another; freedom from the binding influence of the commonplace and hackneyed.

8. A mathematical memory.

9. An ability for spatial concepts.

Points 5, 6, and 7, in particular, are closely allied to the characteristics of the creative mind (see section 1.2).

Krutetskii points out that although some psychologists have stated that there is little in common between abilities for school mathematics and abilities for mathematical activity proper, "it is important to note that in all these instances the psychologists have in mind the usual school mastery of mathematics, unrelated either to heuristics or to the elements of creativity." And he concludes that "a thorough, independent, and creative study of mathematics is a prerequisite for developing an ability for creative mathematical activity--for the independent formulation and solution of problems that have new and socially significant content."[24]

2.2 Creativity and Socioeconomically Disadvantaged Youth

If we agree that creativity is an important aspect of mathematical talent, then we note with great interest the reports of Paul Torrance claiming that youth from socioeconomically disadvantaged cultures excel on scales of creativity in relation to their counterparts from affluent situations.

We have referred to Paul Torrance's follow-up research after Getzels and Jackson's study of creativity and giftedness. Torrance has been particularly interested in the creativity of 'culturally different' youth in the United States, particularly in youth from cultures where poverty is dominant. He has developed tests of creative thinking (The Torrance Tests of Creative Thinking, 1966, 1971, and 1974) which he claims lack cultural bias and thereby allow us to discriminate levels of creative thinking ability and not knowledge of the American mainstream culture.[25]

(Torrance also attributes an absence of cultural bias to the Taylor and Ellison AlphaBiographical Inventory (1966). He has found that this test measures no race difference on the creativity index and a very small race difference on the academic index.)

The Torrance Tests of Creativity, Figural and Verbal Forms A and B, identify as gifted a somewhat different group than those identified by IQ measure. Torrance states that the top 20% on IQ excludes 70% of the top 20% on creativity.[26]

In defense of his statement that the tests lack cultural bias, 86% of the comparisons in race and socioeconomic difference either show 'no difference' or show a difference in favor of the 'culturally different.'

Some might be inclined to judge that if the tests show a difference in favor of the culturally different and the socioeconomically disadvantaged, then they are culturally biased tests but with the opposite of the usual bias. Torrance does not believe that this is so. He believes that culturally different groups encourage in their members some of the more exotic factors of intellect to a much greater extent than does the mainstream culture (many of which are not ordinarily tested on an IQ test), e.g., creative-productive thinking, communication, forecasting, decision making, psychic abilities, extrasensory abilities, flexibility, figural fluency.[27]

Speaking at the Conference on Minority and Disadvantaged Gifted Education in Washington, D.C., in 1978, Torrance claimed that minority and disadvantaged children definitely excel in measures of creativity. He stated that in 'contests' of divergent thinking skills between minority disadvantaged children and their counterparts in the mainstream culture, minority and disadvantaged youth were notably more productive.

The absence of expensive toys and play materials is thought to contribute to skill in improvising with common materials. The lifestyles of economically disadvantaged families develop skills in group activities and problem solving. The value the families place on music, dance, body-expressiveness, and humor keep alive and growing some abilities and sensitivity that tend to die in the children of more affluent families.[28]

We must adapt our teaching style to the strengths of our students. A major strength of culturally different groups arises from the emphasis in those cultures on interdependent, cooperative behavior.[29] Thus, while the mainstream culture emphasises competition, and our schools, in an effort to 'prepare the child for life,' encourage a high degree of competitiveness, the minority child may find such behavior to be quite foreign to his own way of life; thus, he profits less from the educational experience than if he were encouraged to share his special strengths with others. We are sometimes told that, nonetheless, since cutthroat competition is the way of life in the culture in which our student wants eventually to succeed, allowing him to develop his talents in an environment of cooperation will ultimately lead to his destruction; this argument rests on the belief that the student will be unable to cope with the competitive culture after education in a cooperative setting. In response, it must be said that our student will have little skill or talent to use in achieving success if we refuse to provide a school environment in which his talents can develop to their fullest potential. If upon arriving at adulthood, he chooses to avoid the competitive scene, he may do so; there are lifestyle options open to him where his talents may be put to full use in a non-competitive environment.

On the other hand, if we prepare him for the competitive culture at the expense of educating him to much less than his full potential of intellectual achievement, then we have not, after all, prepared him to compete. More than familiarity with competition (along with which usually comes practice in failure), to succeed we need skills, talents, knowledge, creativity; it is for the development of these that schools should exist.

Most significantly, we should teach in a way that emphasizes and strengthens the creative abilities of our creatively gifted students. Torrance has identified a set of creative characteristics that occur with high frequency among disadvantaged children, two of which are relevant to the mathematics classroom: 1) high creative productivity in small groups; and 2) adeptness in visual art activities. But of course, in each learning group, we should be aware of the specific gifts and talents of the students present. That is the only way to optimize the results of our teaching. The possibilities are exciting for both the teacher and the student. If a variety of talents are tested and educated, a student will learn enough about himself to become self-directed. He can direct himself into activities all his life that require his best talents; such a course should lead to optimum productivity and to optimum self-actualization.

Where creativity is the greatest strength in the class, it is especially advantageous to view the students as 'thinkers' and not merely as 'learners.' Jerome Bruner wrote that "thinking is the reward for learning, and we may be systematically depriving our students of this reward as far as school is concerned."[30]

The following list of attributes of the creative thinker will help us to imagine ways of strengthening creativity by capitalizing on its symptoms:

(i) always puzzled about something and seeking answers;
(ii) likely to attempt difficult and dangerous tasks;
(iii) often becomes absorbed in his thinking;
(iv) honest;
(v) may appear to lack consideration for others--creative thinkers may devote their lives for good of others but become so absorbed in problem solving that they may neglect to be polite.[31]

2.3 Mathematics and Socioeconomically Disadvantaged Youth

Are socioeconomically disadvantaged societies likely to produce fewer, or no, mathematically talented individuals?[32] It is at least commonly supposed that mathematics and mathematics-related academic areas are not frequently pursued by individuals whose early years were in a socioeconomically deprived milieu. A major reason for this may be that such individuals do not perceive mathematics as a means by which they can make a major contribution to their society. A child with superior talents who grows up in a deprived situation is likely to be attracted by ideas and vocations that he perceives as able to make significant contributions to his society. (Fox *et al.* have found this to be generally the case with mathematically talented girls; girls are much more attracted to careers that appeal to a desire to give, the socially 'helping' vocations.[33])

Children's interests and abilities are known, too, to be very affected by adult expectations. If we have decided not to expect to discover mathematical talent in certain circumstances, we are likely to discover very little of it.

There are some strange cases in the world of cultural determining of expectation. In South Africa, for instance, the Bantu Education Act legislated that African children should be taught only the rudiments of arithmetic since African children were (by nature or by law?) incapable of learning advanced mathematics!

In fact, the socioeconomically deprived child, having perhaps had much less early childhood exposure to number ideas beyond merely counting, may enter school at a disadvantage to his first grade counterparts in the middleclass (and elsewhere). Furthermore, his schools are less likely to have concrete material to aid in developing his understanding; his teachers are less likely to have the facilities and the time to devote to assuring that he receives experiences that will develop his talent (and his love) for mathematics. Instead, he is quite likely to be given pencil and paper work to do in counting, addition, subtraction, multiplication, and division until the sight of a number on a page is enough to produce intense boredom.

A gifted child in his teenage years may still be suffering from the deprivations of his childhood. It may be that the child who had very little adult attention when he was developing his number concepts may still have questions or doubts about the concepts. The child whose home and school environment provided him with too few concrete experiences in which to test his accumulating abstract knowledge may still feel insecure about basic concepts even though he has progressed far beyond them.

Thus, we must be willing to provide very elementary experiences as well as progressing with the student on the level he has by now attained.

Similarly, due to the tremendous disadvantage that may result from poor language skills, we must be prepared to develop those language skills that the student needs for mathematics. Mathematics is a language--a non-sexist, multicultural language at that--the understanding of which depends largely on the application of logic. A student who has facility in *any* language, standard or non-standard, should not be at a disadvantage in learning to use the language of mathematics. However, the student who has, by the various conflicting forces of his environment, failed to learn any language well will certainly suffer.

For the student for whom logic is not a problem, we still have the job of interpreting mathematics both to and from standard English. Facility in language cannot be divorced from advanced learning in mathematics, and we must make our students aware of that and give them opportunities for improving their skills. Our students must have reading and writing experiences in a mathematical context.

We conclude that socioeconomic disadvantages do affect the learning of mathematics adversely, but that these disadvantages are counterbalanced by other cultural phenomena as discussed above (in section 2.2).

3. Mathematics, Creativity, and the Socioeconomically Disadvantaged

The perils to interest in mathematics, the ability to enjoy it, and the development of talent in it, are numerous and not easily avoided in the environment of the economically deprived child; nonetheless, many students do arrive at their early teens with their love for mathematics intact and their talent still growing and developing. In others, interest can be revived, and in others talent uncovered.

The high school years are crucial if our mathematically talented students are to develop their potential and use it as adults. It is not too late to compensate for deficiencies suffered in the early years of school, and for many people it is the only opportunity to learn much elementary mathematics (basic set theory, computational algebra, elementary geometry, elementary function theory). Very few people ever again have sufficient time to devote to learning elementary mathematics after the high school years.

To be sure, there are individuals so gifted that they succeed (mathematically and otherwise) whether we succeed in contributing to their education or not; we are, however, increasingly being stripped of the notion that a bright mind will make its own way. On the contrary, intellectual talent cannot survive educational neglect and apathy.

Our loss of creative talent is particularly evident in the minority groups, who have in both social and educational environments every pattern of circumstances calculated to stifle talent.[34]

3.1 Recognition

Our first task is to recognize the gifted and talented student so that we can design for him the most suitable educational experience. We need everyone's input for this task. Who is most likely to know of a young person's outstanding mathematical talent? His former mathematics teachers? His classmates? His present teachers? His parents? The student himself? We need input from all of these people. We can also compile a list of names of students who have high scores on the mathematics section of standardized achievement tests given routinely in the schools such as the Comprehensive Test of Basic Skills.

Once a student's name has been suggested, we can gather more information that will help us to judge the degree of his mathematical giftedness:

his mathematics grades
his achievement test scores
scores of any other standardized tests, such as IQ tests
the opinion of his teachers
the opinion of his parents
results of a test of creativity, such as the Torrance tests
a statement of the student's interest in mathematics or math-related activity.

All of these items help in judging the student's talent, but some are considerably more helpful than others. Grades in mathematics courses are sometimes close to meaningless. High achievement test scores are very likely to indicate talent, but low achievement test scores may only indicate something like illness or poor testing conditions. Furthermore, the probability that achievement test data will be available for every student whose records you investigate is somewhat less than 1.

Scores of IQ tests are not routinely available; however, where they are available, they are a valuable indicator of talent. Again, a high score is very likely to indicate high intelligence, but an average score should not be taken as evidence that the student is not gifted. (See section 1.2)

The opinion of his former teachers is useful if obtainable; a well-designed questionnaire will help to obtain this information. However, we must bear in mind that a highly creative child frequently fails to endear himself to his teachers; thus, adverse comments from teachers should not weigh too heavily. Furthermore, we are all aware of studies in which a teacher's expectations of a child have been influenced by information about the child's abilities not based on fact, yet the child's academic performance has altered radically to meet the teacher's expectations.[35]

Parent opinion is invaluable. The parent may be the only person who knows that a child with great talent is sitting quietly unnoticed, or that a real classroom troublemaker learned to read at the age of three, or to count to 100 at the age of two.

High creativity along with the student's expression of avid interest in mathematics is an almost unbeatable combination. Remember that a test of creativity *is* a test of intellectual power. A valid distinction exists between the cognitive function designated as 'creativity' and the *traditional* concept of intelligence, but the traditional concept is too restrictive. Research indicates that the highly creative individual engaged in work or study that interests him highly is a superior achiever.[36]

We must also warn here that, our uses of each notwithstanding, just as a score on a test of intelligence is not 'intelligence,' a score on a test of creativity is not creativity. A poor score need not end your interest in a child's creativity if you think you see some.

Alexinia Baldwin of SUNY has researched the question of identification of culturally different gifted children. The Baldwin Identification Matrix provides a useful form for gathering and evaluating data contributing to identification.

3.2 Enhancing Special Attributes

Once we have identified students we believe are capable of outstanding achievement in mathematics in a school or a school system whose students are primarily from socioeconomically disadvantaged backgrounds, what considerations will help us design a program suitable to meet their mathematical needs?

Bringing these students together is the first big step. Gifted and highly motivated students learn from each other, reinforce each other, and help each other over difficulties. A skillful teacher can provide direction and monitor progress to ascertain that skills are learned as well as theory while creating an atmosphere in which students inspire each other to learn more and at a faster pace than a teacher would be able to do.

In considering the design of the program, let us ask what attributes are common to the subculture of the socioeconomically disadvantaged that enhance an individual's potential to exploit his mathematical talent. We list only three; other special attributes in particular situations should be considered in building a program for that situation.

1. Creativity
2. Facility for working cooperatively
3. Motivation to contribute to the welfare of society (his own society or mankind)

To enhance creativity, encourage students to *use* the creativity they have. Emphasize mathematics as a process so that a student recognizes his own talent for mathematics much as he would recognize a talent for music or art. Encourage active participation on every level in mathematical art, developing experiments and exhibits, computing (with an emphasis on interesting problems), and number theory topics.

Since the creative student is a thinker and a problem solver, the program should not be developed merely with a view to maximization of content coverage, but rather with a view to developing the thinking skills and the outstanding problem solving potential of the students. (More will be said later about good mathematics material and ideas for developing thinking and problem solving skills.) In presenting this kind of experience to young people, the teacher must exercise care not to snatch away all the joy by doing the problems himself, or saying too much, leaving too little to be discovered by problem solving.

When students experience an emphasis on problem solving for the first time, even the potentially strongest problem solvers may be frightened. We must then re-educate them to believe in themselves. To achieve this, we want the student to make a quite different use of his mind than he may have done heretofore in school. "Give students the notion that their minds can be used as instruments."[37] A simple piece of advice for the beginning problem solver from George Polya is that the first requirement of any problem solving sequence is to *get started*. No matter how weak the start, starting increases one's confidence in the ability to carry the task through.

To enhance a facility for working cooperatively, we must be concerned with classroom management. Stress the need for the students to help each other; set as the class goal the maximization of total understanding in the classroom. When working on acquisition of skills, have students work in small groups and make them responsible for each other.

One of the most effective classroom designs for problem solving sessions is to have the students work in pairs. Two people can communicate quickly and effectively; they have the benefit of each other's insight without the time-consuming explanations and discussions that mark the problem solving work of three or four.

Emphasize the value to each person of having as many as possible do well. Even the best students benefit from this by thus gaining the company of other students who can operate on the same plane as they do so that more attention can be devoted to that level by the teacher.

Avoid competition for grades. Students should be aware that the teacher is not the possessor of a certain number of A's, a certain number of B's, etc., which they must compete for. If all students perform at a very high level, all will get a very high grade.

Grading is a controversial practice; does it serve the needs of the student or inhibit them? If a teacher must use grades, or if he chooses to, it may be useful to equalize the "power flow" by having the class grade the teacher on his presentations of new theory or demonstrations of problems. One student, after consultation with the other members of the 'examining committee,' the class, assigns a grade and presents it to the teacher. This can be quite satisfying for the students, and it encourages the teacher when grades are high and provides the immediate negative feedback that is needed if the grade is low.

Jerome Bruner has written:

> One of the most crucial ways in which a culture provides aid in intellectual growth is through a dialogue between the more experienced and the less experienced, providing a means for the internalization of dialogue in thought. The courtesy of conversation may be the main ingredient in the courtesy of teaching.[38]

Such teaching is a goal to strive for in the cooperative classroom.

To appeal to a motivation to contribute to society, we can promote awareness of careers that are mathematics dependent. Very few of our students will be knowledgable about the uses of mathematics; very few of us have a very wide knowledge of its uses. Some of our students may eventually choose mathematics research or teaching, but all of our students need to be aware of the diversity of the fields in which mathematics is an essential ingredient. Inviting professionals who are interested in the students to tell about their work, its social relevance, and its mathematical content, would encourage most of our students.

As well as making known the positive contributions that can be made in the future if the student learns mathematics today, the school should emphasize concern for the community now, and the mathematics class need not divorce itself from these concerns. "Let knowledge as it appears in our schooling be put into the context of action and commitment."[39]

3.3 The Teacher's Role

The teacher's role in a class where creativity and cooperation are emphasized is much less autocratic than is frequently seen in an 'average' classroom. Gifted students frequently provide much of the teaching, and, if the climate of the classroom permits it, will also have input into decisions about course content. Adults who were gifted students frequently express resentment at having been required to put time and energy into a course that interfered with the work they wanted to do on their 'own' problems. But the mathematics of the students' own problems would probably be suitable for course work. Why restrict the course to only traditional course content? Allowing students to choose at least some of the content of the course may lead to the teacher taking the role of research coordinator and learning along with the students. And if that is frightening (they may learn more quickly than their teachers!), do not worry, our students are usually patient with us and rarely give snap quizzes. Our students need models of competence, surely, but they do not need models of omnipotence.

Even when students provide some of the direction for the course, teachers still have the responsibility for coordinating the course, ascertaining that essentials are being covered, and providing appropriate problems and references.

If we provide good text material for our students, then we can use 'lecture time' not in repeating the book's message but in demonstrating the power of an idea, or a technique that results from it, over a particular problem. In the classroom the student should learn more than mathematics; he should learn to mathematize. The key role of the classroom is to stimulate the student to go to his own desk and read, master special techniques and particular details, order his ideas, and tackle some new problems.[40]

3.4 Thoughts About Curriculum

Although gifted students have greater facility for making connections between initially disparate bits of information, and thus are at less of a disadvantage than their more average peers, the more we can make relationships clear by the way we present knowledge and skills, the more effective will be the consequent learning. There is great value in using a unified mathematics curriculum as opposed to the sequence of segmented mathematics courses that have been the traditional curriculum in the United States. (The segmented approach is no longer common outside of the U.S.) Relationships can be made much more obvious when unifying themes (such as functions, vectors, structures) can be seen throughout.

If a student in the traditional curriculum discontinues his study of mathematics at any point before calculus, he will have acquired virtually no mathematics applicable to work he may wish to pursue; a student who has studied mathematics in a modern unified curriculum will have seen relationships and concepts that he can apply practically in his work at whatever point he withdraws from study. The student who emerges from the traditional curriculum to pursue studies in biology, economics, linguistics, psychology, will find himself confronted with a completely new kind of mathematics; he will have had no experience in developing concepts and structures, and faced with the need to mathematize a situation, he may be quite unskilled.

For the student who continues his study of mathematics, the unifying themes make relationships so much clearer and problem solution so much more accessible that progress can be more efficient and more effective.

The attempt of the traditional mathematics curriculum to segment mathematics learning into a course in geometry and two courses in algebra and to keep those two worlds apart distorts the later learning of the student of mathematics. Geometry is a world of pictures. It casts a light that illuminates our work and gives it beauty. It guides us in our search for truth and suggests new directions and new problems. Algebra is a world of symbols. The light it casts is hard and sure. It allows us to pin down our ideas and make our truths precise. It has its patterns, but they are harder to perceive. Each world has its own language; the mathematics student must learn to translate freely from one to the other so that he can use the grammar of one to gain insights into the structure of the other.[41]

It is also extremely desirable to coordinate mathematics learning with learning in other subject areas. Learning mathematics through problem solving can be effected through problems that arise in physics or biology or social studies. The integration of the history of mathematics and science with political and social history can be effective and can inspire greater interest in history as well as deeper knowledge of mathematics. At least some of our students have special needs in the area of language development, and this should be integrated with their most avid interest; hence the gifted mathematics student, as well as reading good English literature, should be encouraged to read mathematics and to write about what he has read.

An integrated curriculum requires a great deal of time and care and coordination and planning, but the returns are enormous. Whatever degree of success we can achieve in integrating subject areas, we can at least put our own house in order and provide a unified, integrated mathematics curriculum for our students.

It is possible to take a unified approach to mathematics using traditional textbooks if these are the only ones that are available. There are also at least three series of unified approach texts published in the United States that can be used satisfactorily.

Unified Mathematics courses 1-6 by Fehr, Fey, and Hill, published by Addison-Wesley and the Columbia University Press, are books intended for very bright children beginning in Grade Seven. These books contain challenging problem sets emphasizing problem solving skills and learning theory by development through problems. They are also useful for an overview in determining course content if it is necessary to use books that the school already has on hand.

The Comprehensive School Mathematics Program's *Elements of Mathematics*, published by CEMREL of St. Ann, Missouri, is another program for advanced classes, somewhat different from the Fehr, Fey, and Hill series. (CEMREL's elementary school program is designed for all students; their secondary program, however, is not meant for the average student.) *Elements of Mathematics* emphasizes challenging problems, mathematization of 'real' situations, unifying concepts, and careful mathematical linguistics.

A third unified mathematics program published in the United States is a three year program for average students published by Charles Merrill. It is not relevant to our present discussion but would be a great step forward for our average students.

Various unified mathematics curriculum projects from Britain are also available in the United States. Of these, the School Mathematics Project is perhaps the most readily available. The South Nottinghamshire Project, published by Blackie, has excellent material.

We should choose our basic texts according to what we hope to teach our students. We want to develop their creative problem solving abilities, give them the skill to solve problems not necessarily set in mathematical terms, expose them to the beauty of the structure of mathematics, and prepare them to learn more mathematics. The texts we choose for our students should help promote these aims.

Too often we 'teach' students all the interesting aspects of a theory and then ask them to apply the results; worse yet, we may give them an example of how to apply the results and all we ask of them is to follow our example. Dependence on conventional textbooks encourages such teaching; a teacher may be swept along before taking time to reflect on what is happening. Students need questions that leave the development of some of the theory to them and serve as starting points for further study. Much of this should be developed by the teacher working with bright and turned-on mathematics students and exchanged with other teachers doing the same thing. For excellent examples and ideas for applying them, see *Starting Points* by Banwell, Saunders, and Tahta.

The *Network* books (*Network: A Mathematics Series* published by Hutchinson for Leapfrogs) are a series of books with a wide range of mathematical ideas for middle and secondary schools. They include three groups of books: "Action Books" which encourage mathematical exploration; "Link Books," anthologies of pictures and diagrams. Some pages invite mathematical activity, some pose problems, some illustrate ideas, some provoke thought. The title page of one of these looks something like this:

LINKS

A BOOK OF PICTURES AND DIAGRAMS
WHICH HAVE BEEN SELECTED
WITH CERTAIN MATHEMATICAL IDEAS
IN MIND
Not to illustrate these ideas
But to
SHOW PARTS OF THE REAL WORLD
FROM WHICH MATHEMATICS COMES

Those who turn the pages of this book
Are invited to reflect on what they find herein
And to pursue such actions as they think appropriate
-Drawing, writing, making models, talking
MAKING THEIR OWN COLLECTIONS OF PICTURES, &c, &c.

The third group, "With Few Words," are workbooks that are almost entirely visual and, with few or no words, direct the reader to some very fruitful mathematical activity.

The books are quite inexpensive, but if their cost remains a problem, one copy provides lots of inspiration. Leapfrogs, the group responsible for these little books, publish many other inspiring little booklets from which we can learn a lot about mathematics and about the way in which mathematical knowledge has been developed. Some of these are listed in the bibliography.

The materials suggested here and their descriptions serve only to give the flavor of the kind of experience that is valuable in the classroom; but each teacher with each class will surely develop new problems and better ideas in an arena of creative activity. The teacher's aim is to impart the information of mathematics at the same time as developing and nurturing the student's inherent creativity, the quality that can set him apart from the average.

3.5 Thoughts on Particular Approaches to Teaching and Learning Mathematics

To go beyond the practice of algorithms and set problems, we try to provide students with applications of the mathematics they have learned. Convincing a student that a 'word' problem in a typical textbook is a real application of mathematics would be a most challenging task.

A practical problem is a problem to which we would really like to know the answer. We *must* provide our students with interesting and challenging problems. "Let the skills of problem solving be given a chance to develop on problems that have an inherent passion."[42]

Good problems are the chief vehicle for good curricula. After investigation of an interesting problem, the individual student, or the group, or the class (whoever worked on the problem) should write up the investigation and the results. It is valuable to describe the process of the investigation as well as the result in the write-up. (For some examples of write-ups done by students, see *Starting Points*, pages 40-47.)

If it is difficult for students to get started on a write-up, the teacher can help by asking questions. "Where did you start?" "What did you do then?" "What did you do when you discovered that?" Write-ups inevitably improve with practice.

After the student has had experience in solving a wide variety of interesting and challenging problems, it is important to look at the question of application from another point of view. Students need to have the experience of mathematizing a 'real' situation at several levels of sophistication. We should explore with our students the problems of constructing a mathematical model and discuss both what we gain and what we lose by making a mathematical abstraction of a situation.

The English Platonist Weldon once said:

> There are three kinds of things in this world: there are troubles which we do not know quite how to handle; then there are puzzles with their clear conditions and unique solutions, marvelously elegant; and then there are problems--and these we invent by finding an appropriate puzzle form to impose upon a trouble.[43]

Course 4 of *Unified Mathematics* discusses mathematical modelling. *Elements of Mathematics* devotes a unit of its introductory section (called Book A) to making a mathematical model of an apparently 'non-mathematical' situation and also offers other problems in mathematization.

Mathematical literacy is essential to a community's survival in the modern age. Knowledge of the calculus is fundamental to this. Traditionally, the first course in calculus is a stumbling block for countless students, and poor preparation is almost certainly the cause of this. All of our pre-calculus mathematics is unified in a common purpose ("At last!" exclaims the student of the traditional segmented curriculum in relief), and certainly inadequate understanding of the Real numbers, or the concept of function, or particular elementary functions would adversely affect the student's success at this stage. But another source of the difficulty encountered at this stage is mathematical *language*. We would suggest that the language used in all our mathematics courses should be precise and carefully chosen. The ideas of, for example, convergence and continuity require a fair degree of sophistication in the use of language. A facility with the use of quantifiers is essential if the student is to feel comfortable with a phrase such as 'for every ε there exists δ such that....' Practice in the use of quantifiers should begin in much less sophisticated settings than in statements of definitions of limit and continuity of a function. The student should be introduced to the beauty and precision of correct mathematical language much earlier than traditionally happens. "Many of the traditional difficulties with calculus will vanish if students come to it with linguistic habits of the right sort."[44]

Let us hasten to add that in our concern for precision and correctness in mathematical language we must not go overboard in the proliferation of mathematical notation. Students should see and become acquainted with standard mathematical notation, but their success in mathematics should never be determined by the degree of comfort they attain with the symbols of mathematical logic. Mathematics is not notation; it is a set of ideas.

In the torrid debate over acceleration vs. enrichment, we can take neither side. Certainly a gifted student can learn mathematics (and much else) at a much faster rate than that provided for the average student; to hold him to that rate is often to stifle his enjoyment of learning. But the traditional curriculum is quite unsatisfactory in content; the gifted student needs much broader mathematical experiences and opportunities to learn in much greater depth than are traditionally offered. Some of these differences are provided for in the curriculum and in the challenging problems we provide; others are inherent in a different approach to teaching and learning.

Epilogue

Many questions remain unanswered in the area defined by the intersection of creative talent, mathematics, and socioeconomic disadvantage, and many assumptions remain unverified.

(1) It seems highly probable that high creativity coupled with high interest in mathematics will result in outstanding mathematics achievement. Observation bears this out, but no formal studies are available.

(2) Do scores on tests of creativity correlate with mathematical creativity? Can we devise measures that have a very high correlation with observed mathematical creativity?

(3) Is it true that active cooperation in classrooms of minority and disadvantaged children produce higher mathematics achievement than an atmosphere of competition? Where is this true? Where is it false? Are some aspects of competition helpful?

(4) What are the most effective modes of classroom management to promote mathematics learning through cooperation. What particular teaching techniques best enhance the cooperative spirit present in the classroom?

(5) What teaching techniques best motivate the students for independent learning and problem solving?

A sharing of observations, either formal or informal, on these questions by teachers and other concerned persons would contribute to wider knowledge and help us to develop better learning situations for gifted students. If you are interested in participating in research concerning the mathematically gifted youth in deprived circumstances, or if you have comments on any aspect of this paper, please write to

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